EE565:Mobile Robotics

Welcome

Dr. Ahmad Kamal Nasir
Dr. Abubakr Muhammad

Organization

- **Monday: 1750-1850**
  - Lectures and Discussion
  - Lecturer: Ahmad Kamal Nasir, Abubakr Muhammad
  - My Office hours:
    - Tuesday [1400-1500]
    - Thursday [1400-1500]
- **Wednesday: 1750-1850**
  - Lab course, lab work and home exercises
  - Teaching Assistance: Hamza Anwar, Mudassir Khan
  - TA Office hours:
    - Monday [0000-0000]
    - Friday [0000-0000]

Who are We?

- **Cyphynet labs**
  – 2 Professors, 3 PhD students, 4 Research Associate
- **Research interests**
  – Robotics and Hydro-systems for welfare and sustainable development of Pakistan
- **My research goals**
  – Apply solutions from computer vision and control systems to real world problems in mobile robotics.

Course Objectives

- **Hands-on experience** on real aerial and ground mobile robots.
- Provides an overview of **problems and approaches** in mobile robotics.
- Introducing **probabilistic algorithms** to solve mobile robotics problems.
- Implement state of the art probabilistic algorithms for mobile robots with a strong focus on **vision** as the main sensor.

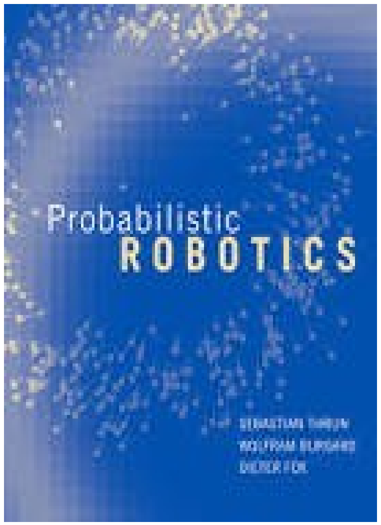
Course Learning Outcomes [CLO]

1. Understand basic wheel **robot kinematics, common mobile robot sensors and actuators** knowledge.
2. Understand and able to apply various robot **motion and sensor models** used for recursive state estimation techniques.
3. Demonstrate **Inertial/visual odometeric** techniques for mobile robots pose calculations.
4. Use and apply any one of the **Simultaneous Localization and Mapping** (SLAM) technique.
5. Understand and apply **path planning and navigation** algorithms.

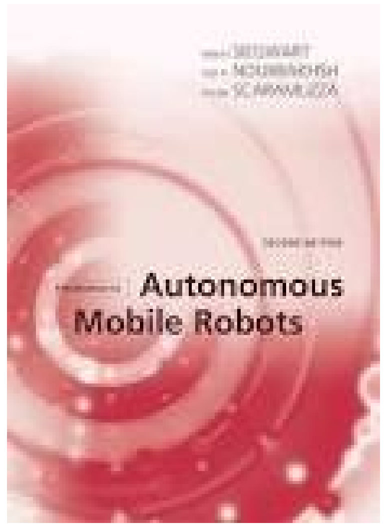
Course Website

- **Course Website**
  - http://lms.lums.edu.pk
  - http://cyphynets.lums.edu.pk/index.php/EE-565
  - Lecture Slides
  - Lab Exercise and Resources
  - Course outline and schedules
  - Announcements
- I need your **feedback** to improve this course
- Let me know in person or by email for improvements and mistakes…

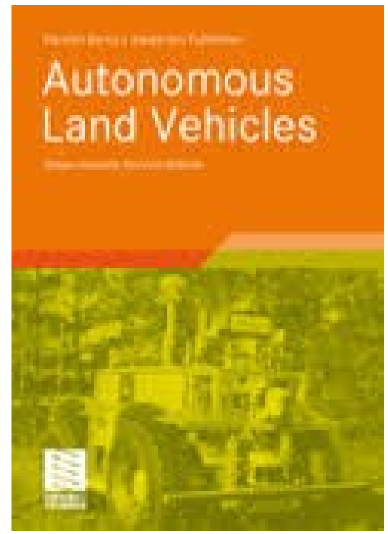
Course Material



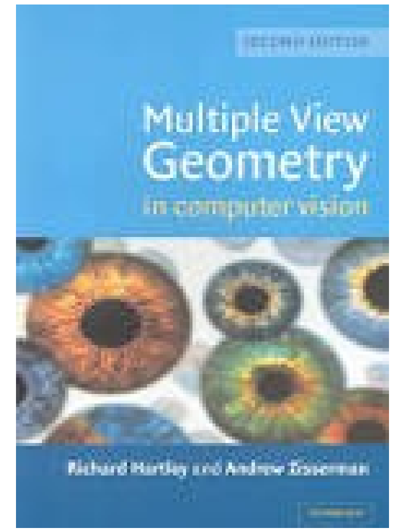
Probabilistic Robotics by *Sebastian Thurn, MIT Press 2005*



Introduction to Autonomous Mobile Robots by *Roland Siegwart , MIT Press , 2004*



Autonomous Land Vehicles by *Karsten Berns , Springer, 2009*



Multiple View Geometry In Computer Vision *by Richard Hartely,* Cambridge University Press, 2004

Week No.	Module	Lecture Topics
1	Mobile Robot Kinematics	Course Introduction and Objectives, Short notes on Linear Algebra, Recap of Probability Rules, 2D/3D Geometry, Transformations, 3D-2D Projections
2		Wheel Kinematics and Robot Pose calculation, Mobile robot sensors and actuators
3	Sensor Fusion and State Estimation	Motion Models (Velocity and Odometry), Sensor Models (Beam, Laser, Kinect, Camera)
4		Recursive State Estimation: Least Square, Bayes Filter,Linear Kalman Filter, Extended Kalman Filter
5*		Non-parametric filters, Histogram filters, Particle filters
6	Inertial and Visual Odometry	Inertial sensors models, Gyroscope, Accelerometer, Magnetometer, GPS, Inertial Odometry, **Mid-Term Examination**
7		Visual Odometry: Camera model, calibration, Feature detection: Harris corners, SIFT/SURF etc., Kanade-Lucas-Tomasi Tracker (Optical Flow)
8		Epi-polar geometry for multi-view Camera motion estimation, Structure From Motion (SFM): Environment mapping (Structure), Robot/Camera pose estimation (Motion)
9	Localization and Mapping	Natural, Artificial and GPS based localization, Kalman Filter based localization, Optical flow based localization
10		Map representation, Feature mapping, Grid Mapping, Introduction to SLAM, Feature/Landmark SLAM, Grid Mapping (GMapping) , **Mid-Term Examination**
11*		RGBD SLAM
12	Navigation and Path Planning	Obstacle avoidance: configuration/work spaces, Bug Algorithm, Path Planning algorithms: Dijkstra, Greedy First, A*
13*		Exploration, Roadmaps
14		Recap, Recent research works and future directions
15		**Final Presentations**

Week No.	Module	Lab Tasks / Tutorials
1	Mobile Robot Kinematics	Introduction to ROS
2		ROS interface with simulation environment
3	Sensor Fusion and State Estimation	ROS Interface with low level control
4		IRobot setup with ROS and implement odometeric motion model
5		AR Drone setup with ROS and Sensor data fusion using AR Drone's accelerometer and gyroscope
6	Inertial and Visual Odometry	**Mid-Term Examination**
7		Inertial Odometry using AR Drone's IMU and calculating measurement's covariance
8		Calibrate AR Drone's camera and perform online optical flow.
9	Localization and Mapping	Using AR Drone's camera, perform visual odometry by SFM algorithm
10		**Mid-Term Examination**
11		Creating grid map using IRobot equipped with laser scanner.
12	Navigation and Path Planning	Create a 3D grid map using IRobot equipped with Microsoft Kinect.
13		Setup and perform navigation using ROS navigation stack and stored map.
14		Hands-on introduction to sampling based planners via Open Motion Planning Library (OMPL)
15		**Final Presentations**

Lab Tasks

- Lab Task Format: Make pair and submit name
- Lab Exercise Deadline: Before next lab
- Submission method: LMS, E-mail
- Instructions for Lab Completion: Manual, TA

Lab Resources

- Four IRobots, Four AR-Drone, Four MS Kinect, One Laser Ranger Scanner
- 26 Students, 2 Studetns/Group
- Sign-up for a team before Lab.
- Either use lab computers or bring your own laptop (**Recommended**)

Lab Safety

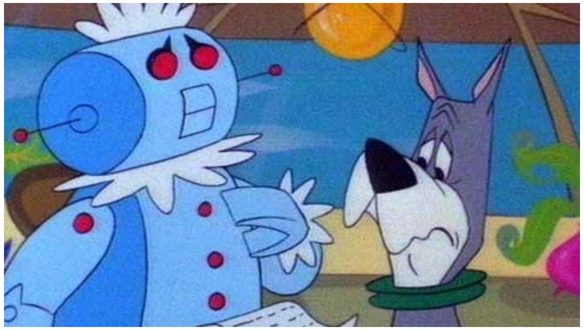
- **Read** Lab manual/instructions before you start
- Be **careful of the moving parts** of the mobile robots.
- Quad-rotors are dangerous objects, Never **touch** the rotating propellers.
- Don't try to **catch** the Quad-rotor when it **fails**, Let it Fall!
- If somebody get **injured** or something get **damaged** report to us.

Today's Objectives:

- Introduction to Mobile Robotics
  - Approaches
  - Trends
- Short notes on linear algebra
- Recap of 2D and 3D Geometry
- Transformations, 3D-2D Projections
- Recap of Probability Rules

Introduction to mobile robotics

- Public perception
- Pop culture images

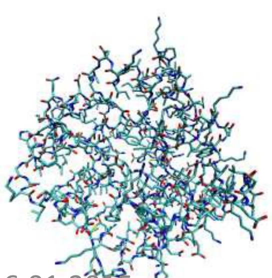


What is a Robot?

A mechanical system that has sensing, actuation and computation capabilities.

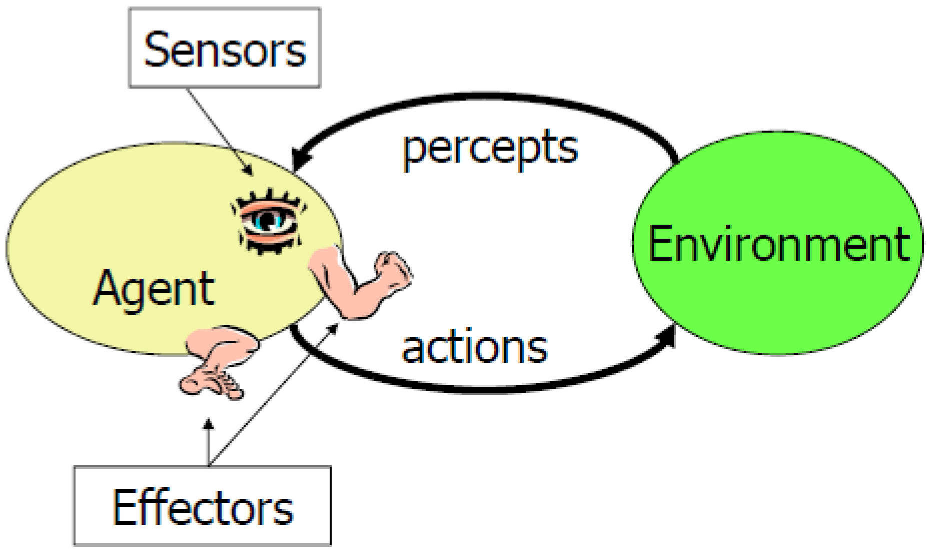
Other names (in other disciplines)

- Autonomous system
- Intelligent agent
- Control system



What Makes a Robot?

- A robot consists of:
  - sensors
  - effectors/actuators
  - communication
  - controller



- A robot is a rational agent capab
  - acting autonomously
  - achieving goals

- *Robota* means self labour, drudgery, hardwork in Czech
- روبالہ = رو +بہ +آلہ (Urdu Wikipedia)

What is Robotics?

- The art and science of making robots

- Where are roboticists found
  - Electrical engineering (control systems)
  - Mechanical engineering (mechanisms)
  - Computer science (AI, learning)
  - Mechatronics
  - Bioengineering

- Increasingly important
  - Lawyers (legal issues, labor laws)
  - Philosophers (ethical issues)
  - Economists (disruptive technologies)
  - Social scientists (the social impacts of automation, aesthetics)

Roboticist and Robot Ethics

- A robot may not injure a human being, or, through inaction, allow a human being to come to harm.
- A robot must obey the orders given it by human beings except when such orders would conflict with the first law.
- A robot must protect its own existence as long as such protection does not conflict with the first or second law.

[Runaround, 1942]

Current Trends In Mobile Robotics

- Robots are moving away from factory floors to
  - Personal Service, Medical Surgery, Industrial Automation (Mining, Harvesting), Hazardous Environment (Space, Underwater) etc.
- Mobile Robots Domains
- Ground Robots
- Flying Robots

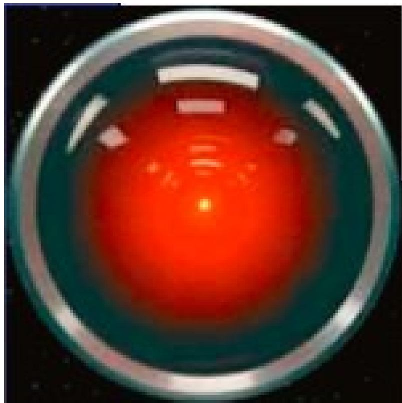
Modern Robotics

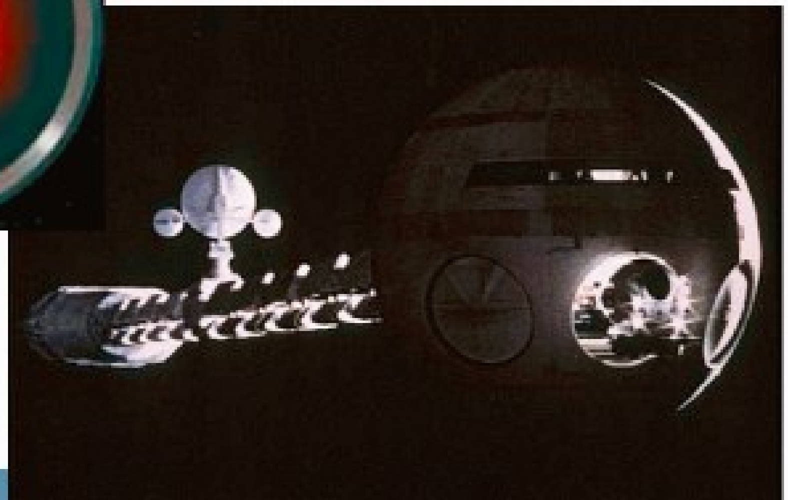
Three broad categories

1. Industrial robots: manipulators (1970's)
2. Mobile robots: platforms with autonomy (1980's)
3. Mobile manipulators = manipulator + mobility (2000's)

Industrial Manipulators

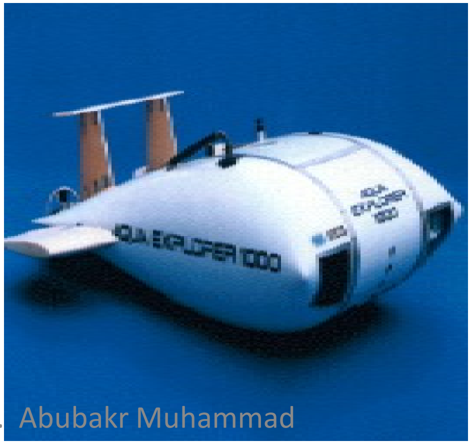
Unmanned Vehicles





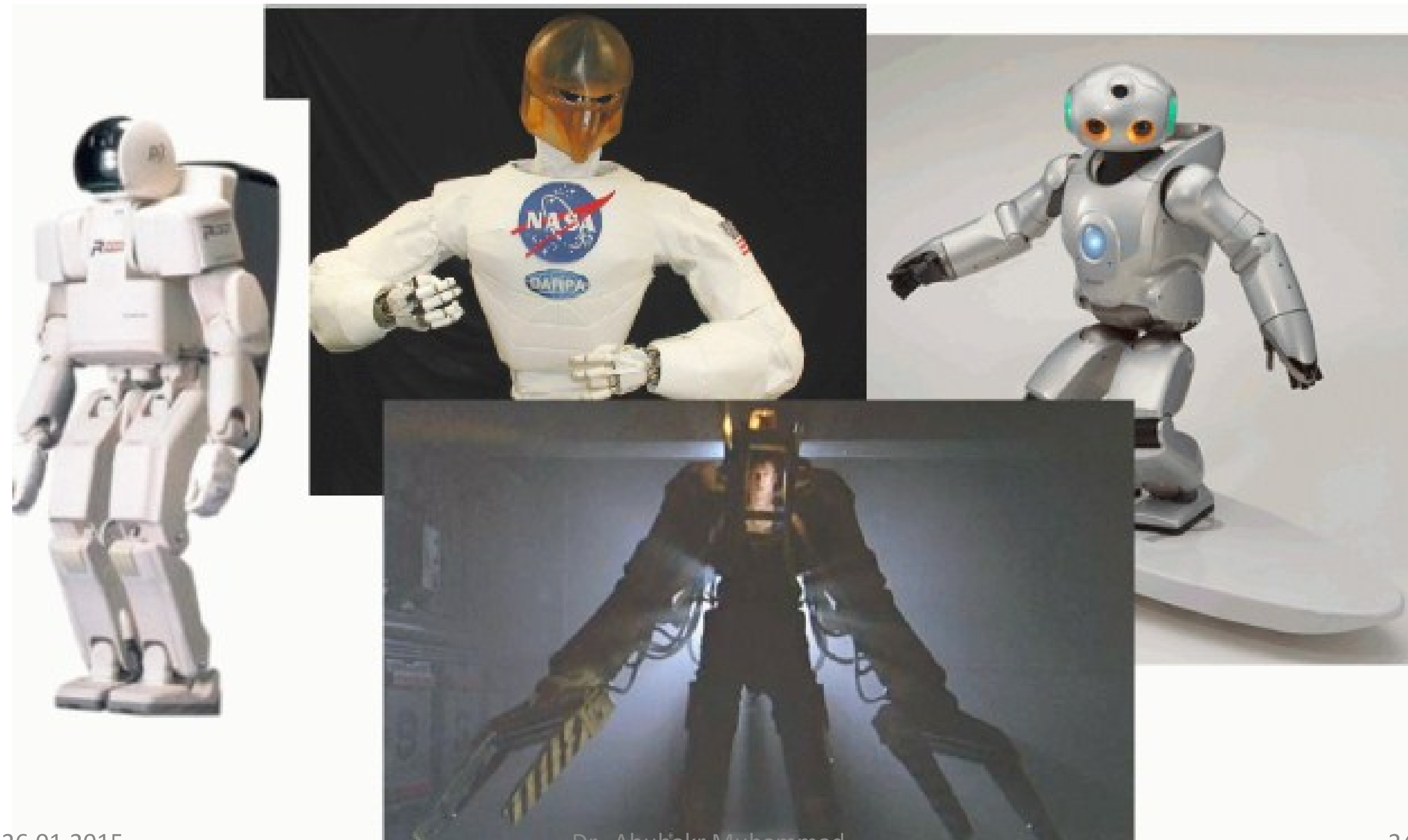
Some Mobile Robots Terminology

- UAV: Unmanned Aerial Vehicle
- UGV: Unmanned Ground Vehicle
- UUV: Unmanned Undersea (underwater) Vehicle
- AUV: Autonomous Underwater Vehicle



Anthropomorphic Robots

(Having human form or attributes)



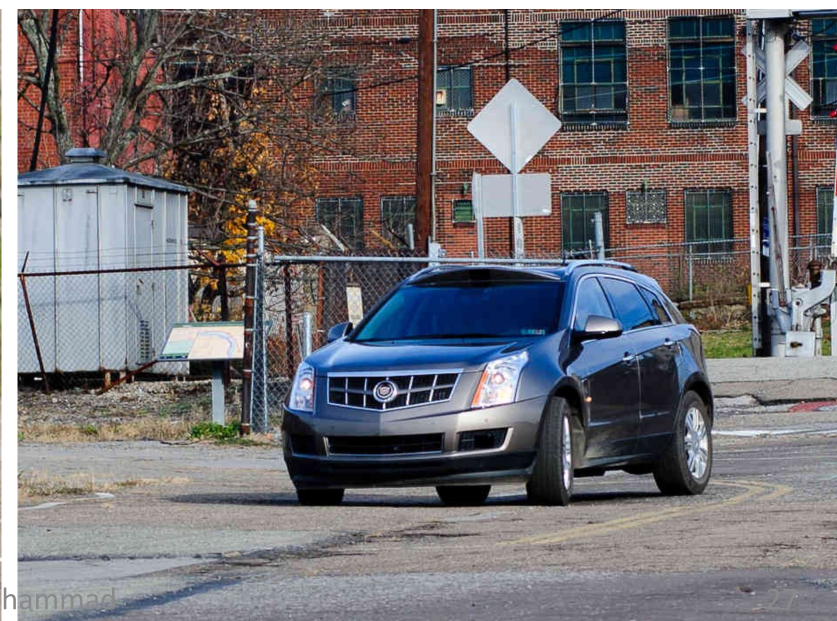
Bio-inspired / Walking Machines

Self-Driving Trucks for Mining

- 17 Self-driving trucks deployed for mining in australia
- Increased accuracy in operation as compared to humans
- Improved earth excavation

Autonomous Driving

- Market for advanced driver assistance systems to grow from **$10 billion** now to **$130 billion** in 2016
- Projected to reach **$500 billion** by 2020

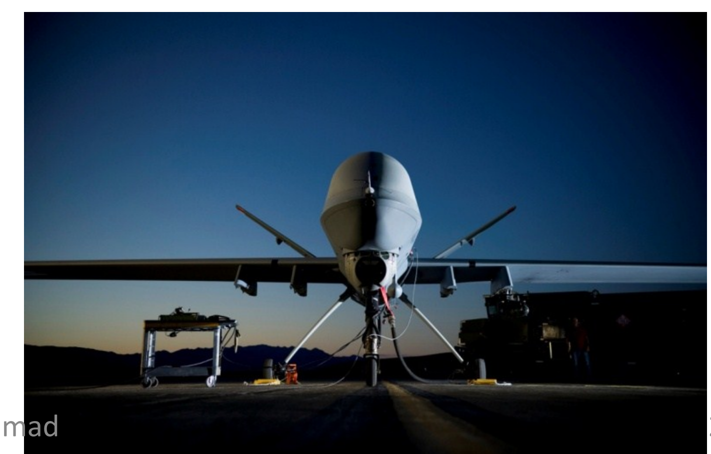


Autonomous Driving

- Tesla—90% autonomous vehicle within 3 years
- EURO-NCAP automated emergency braking mandatory by 2014
- For 5-star safety rating, vehicle has to be 'robotic'

Defense: Unmanned Aerial Vehicles

- Drones—combat, surveillance
- First appeared during the vietnam war
- First recorded targeted killing– 2002 (afghanistan)
- Global UAV market--**$5.9 billion** now to **$8.35 billion** in 2018



Defense: Unmanned Aerial Vehicles

- NYU/stanford report—2,562-3,325 fatalities in pakistan
- U.S pullout from Afghanistan-- integration of decommissioned UAVs
- Market ripe for drones for surveillance
- Other uses: weather research, law enforcement

Defense: Driverless Vehicles

- 1/3 of all U.S Military vehicles to be autonomous by 2015
- *Terramax*-- Oshkosh Trucking Corporation
- *Black Knight*– Unmanned Tank

Unmanned Agricultural Machines

- Efficient utilization of resources
- Uavs for spraying insecticides
- Driverless tractors

Unmanned Agricultural Machines

- Possible Applications: Weeding, Harvesting, Pruning, Canal Cleaning (‘*Bhal Safai*’)
- Lettuce Bot (Blue River Technology)—Eliminates Leafy Buds **20x** Faster

Humanitarian

- Landmine detection
- Bomb disposal
- Prosthetic limbs—full restoration of original capabilities

Surgical Robots

- Surgical robotics-higher precision, repeatability, cost-effective
- Significantly lower blood loss
- Minimally invasive surgery

Surgical Robots

- Flagship--da vinci surgical robot
- Surgical robot market to reach significant growth
- Market size: **\$3.2 billion** in 2012, anticipated to reach **\$19.96 billion** by 2019

Assistive Robots

- Robotic vacuum cleaners
- Global market share of robotic vacuum cleaners-- **12%** of **$680 million**

Assistive Robotics

- Growing elderly population in developed countries
- Demographics to change by 2050
- Over 60 to form **22%** of the world population compared to the **11%** today
- Needs: visual assistance, emergency assistance, mobility assistance

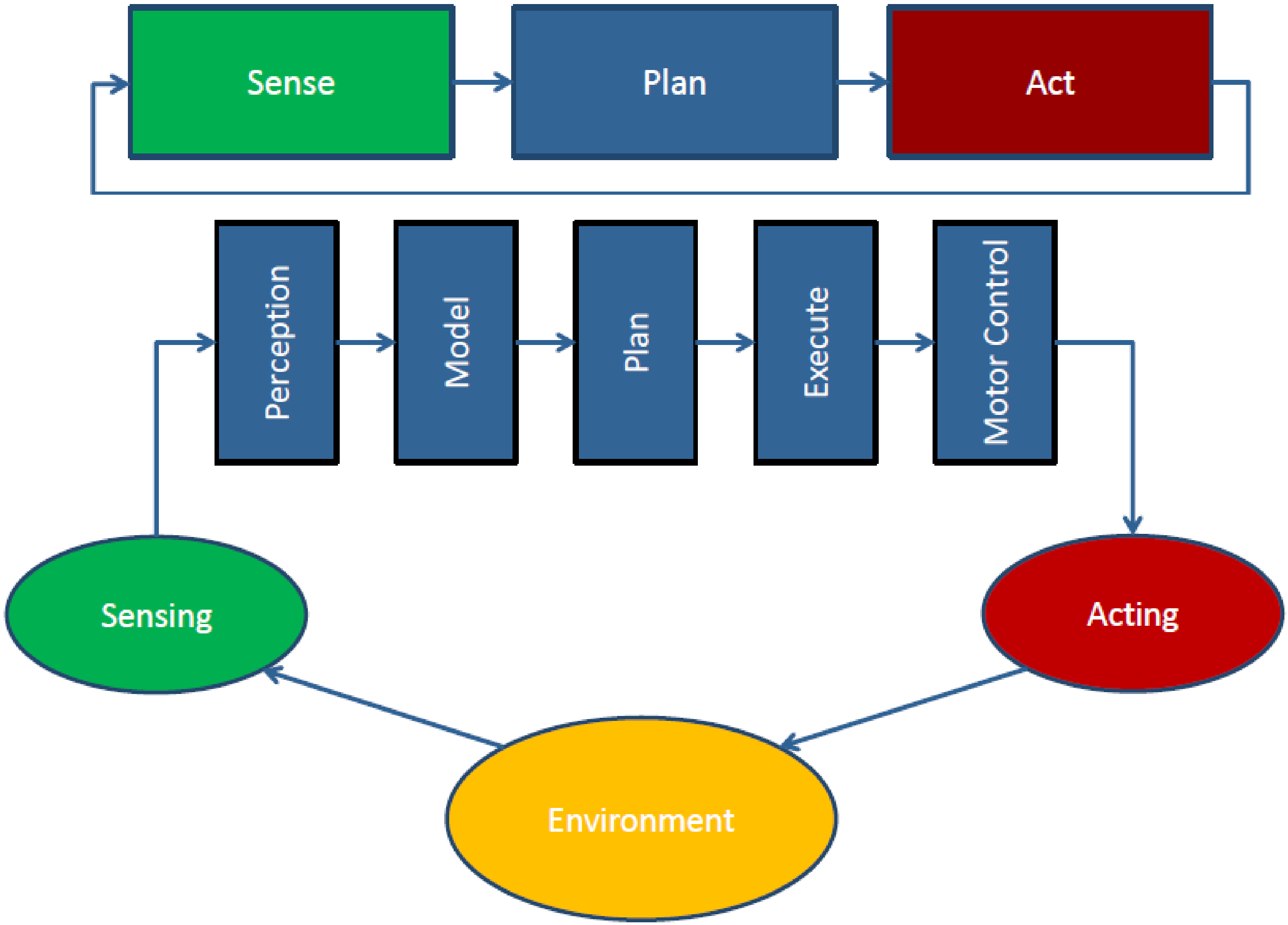
Factories of the Future

- Declining costs
  - Industrial grade manipulators ~ > $100,000
  - Baxter (rethink robotics) costs **$22,000**
- Small & Medium Enterprises (SME's) entering the fray
- Need consistent quality
- Lean operation
- Higher productivity
- Higher accuracy in safety critical applications

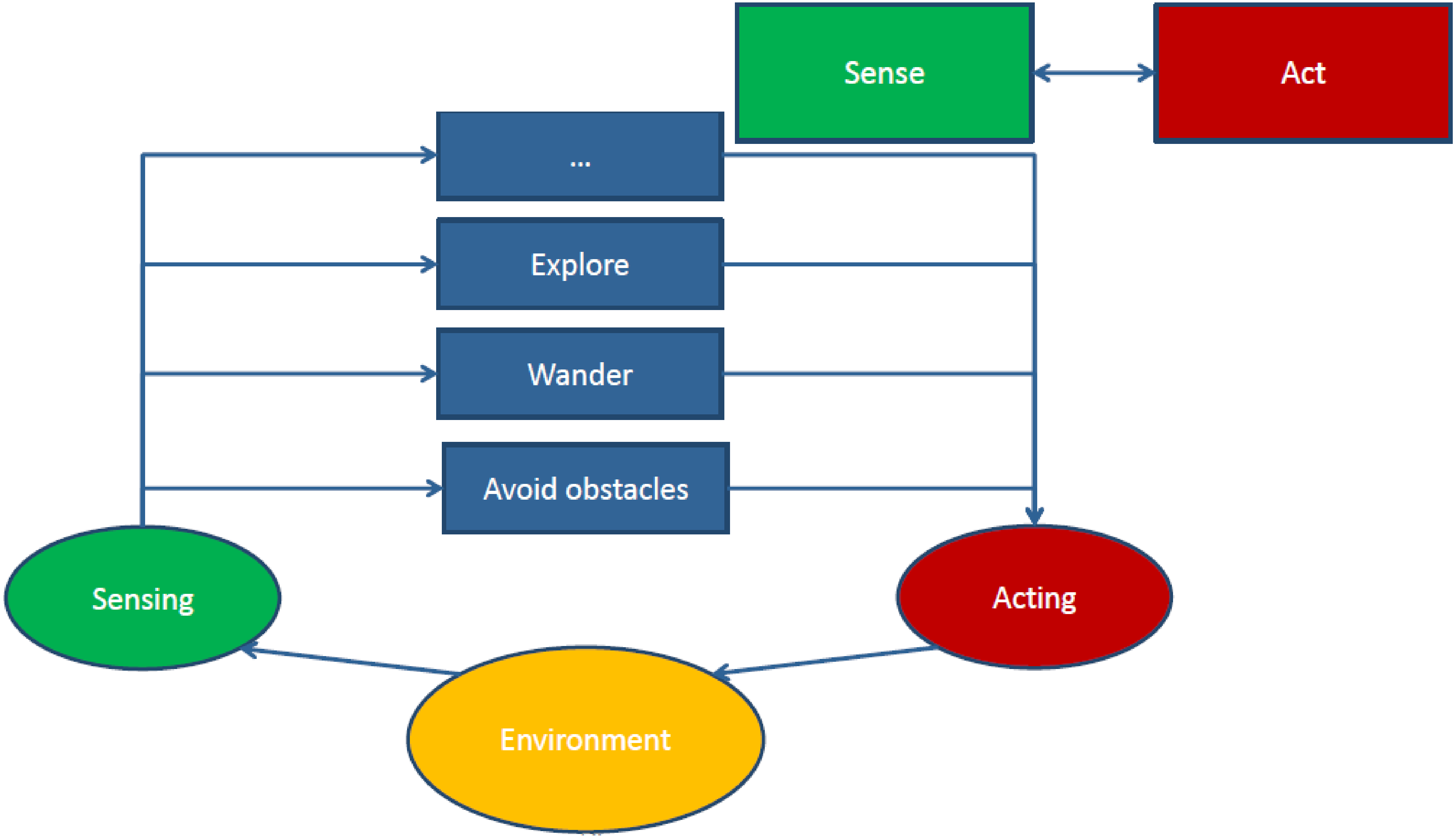
Paradigms in Robotics

- Classical, until 1980
- Reactive, until 2000
  - Behavior Based
  - Hybrid
- Probabilistic, present

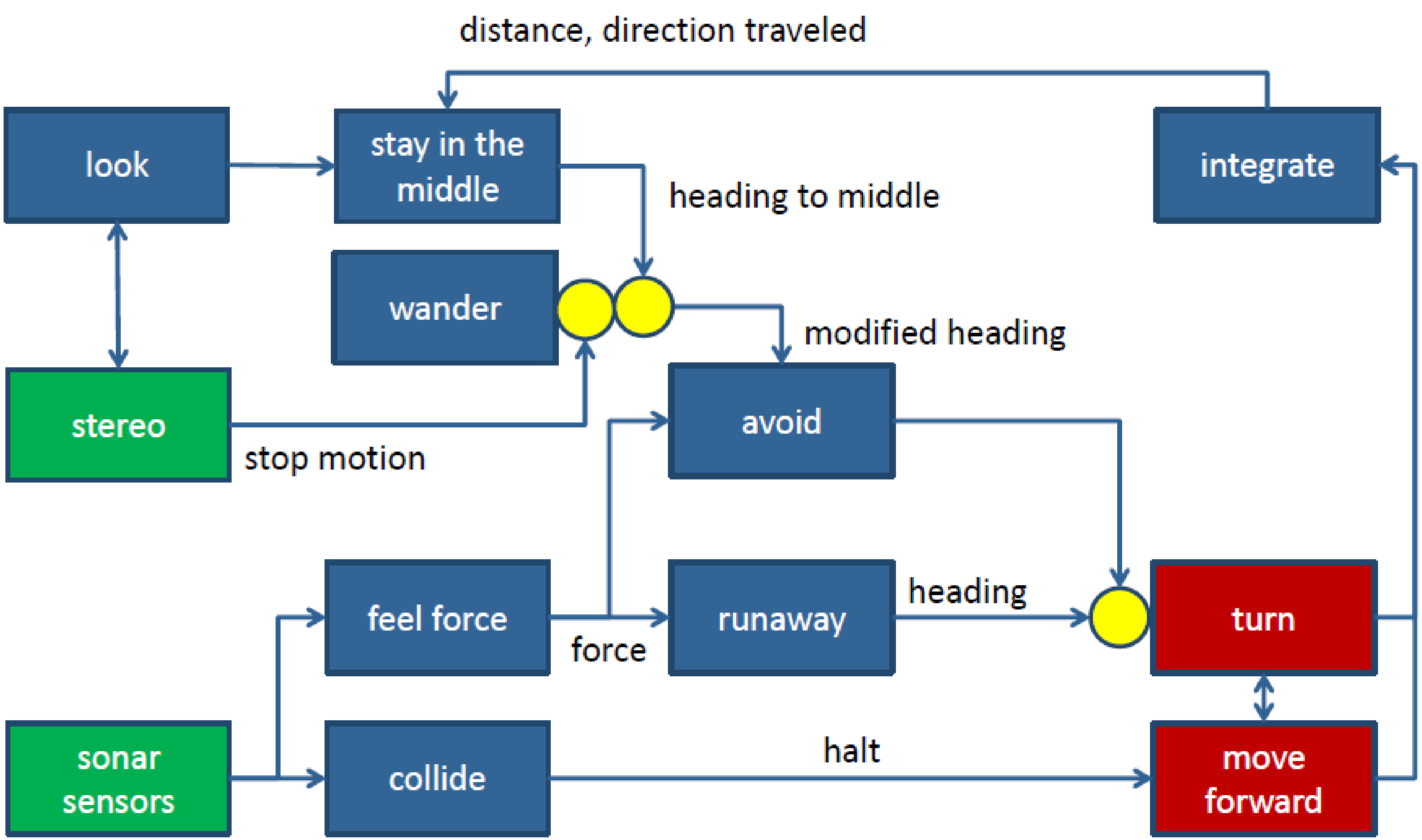
Classical/hierarchical



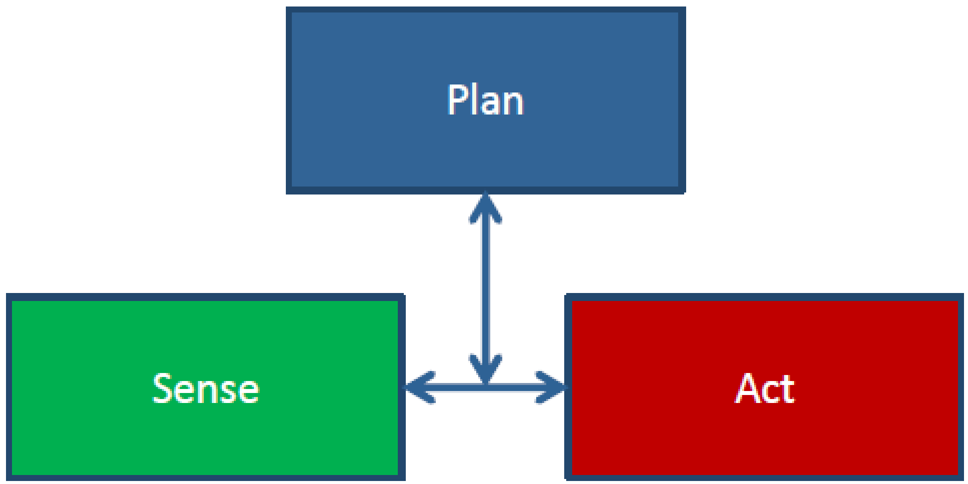
Reactive Paradigm



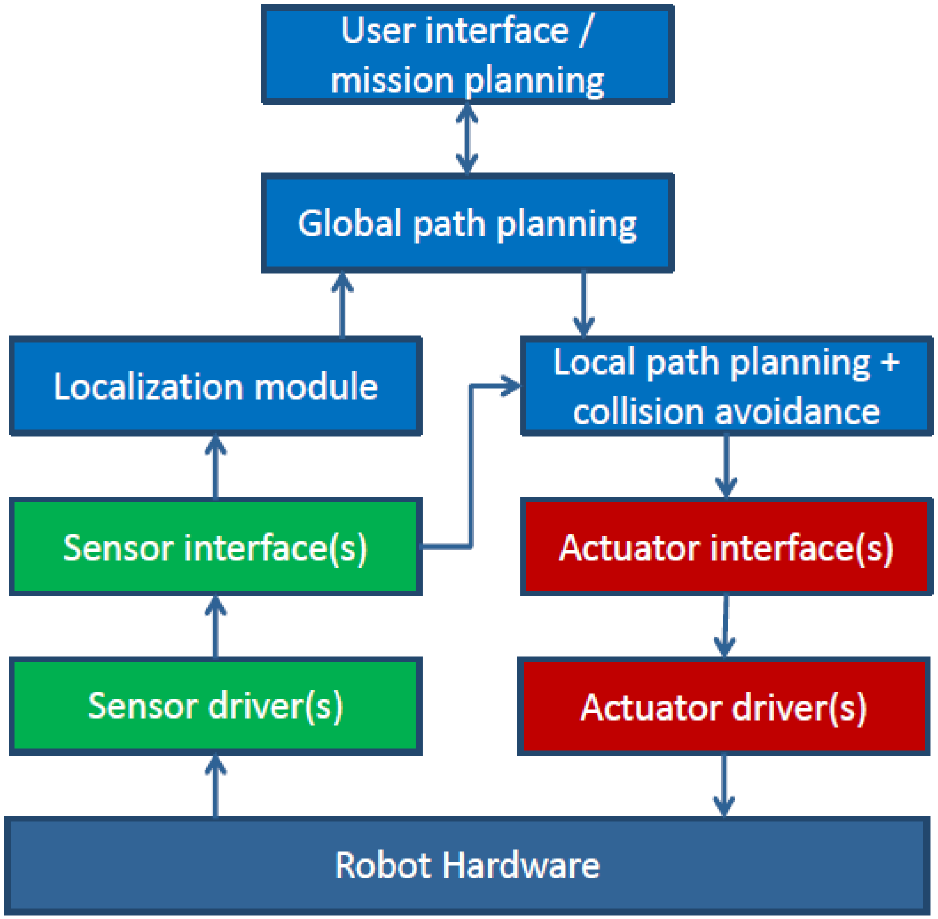
Behaviors Based Robotics

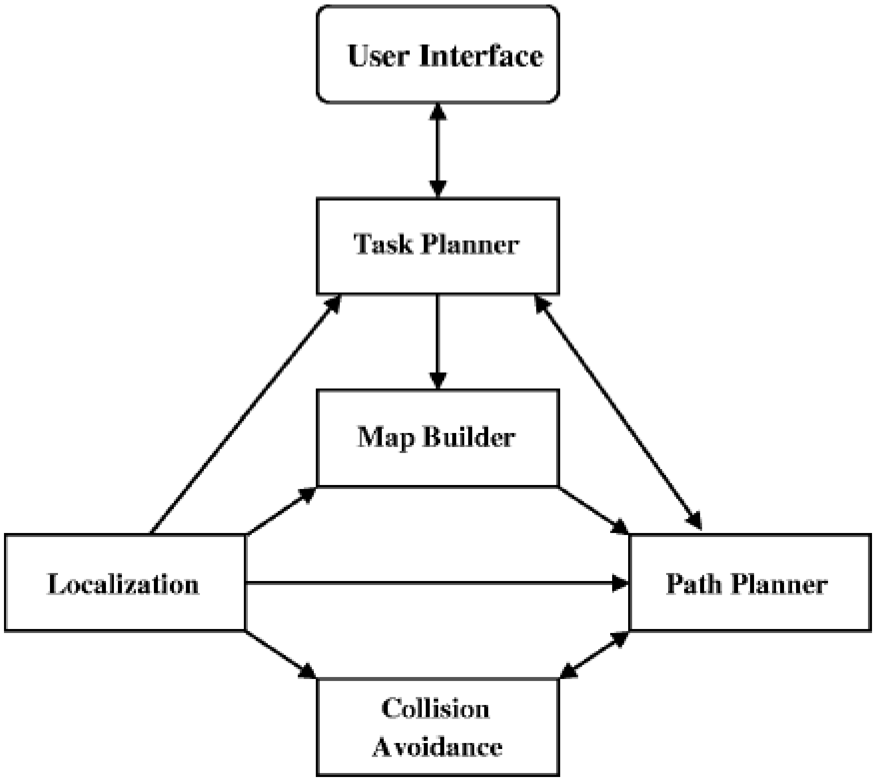


Hybrid deliberative/reactive Paradigm

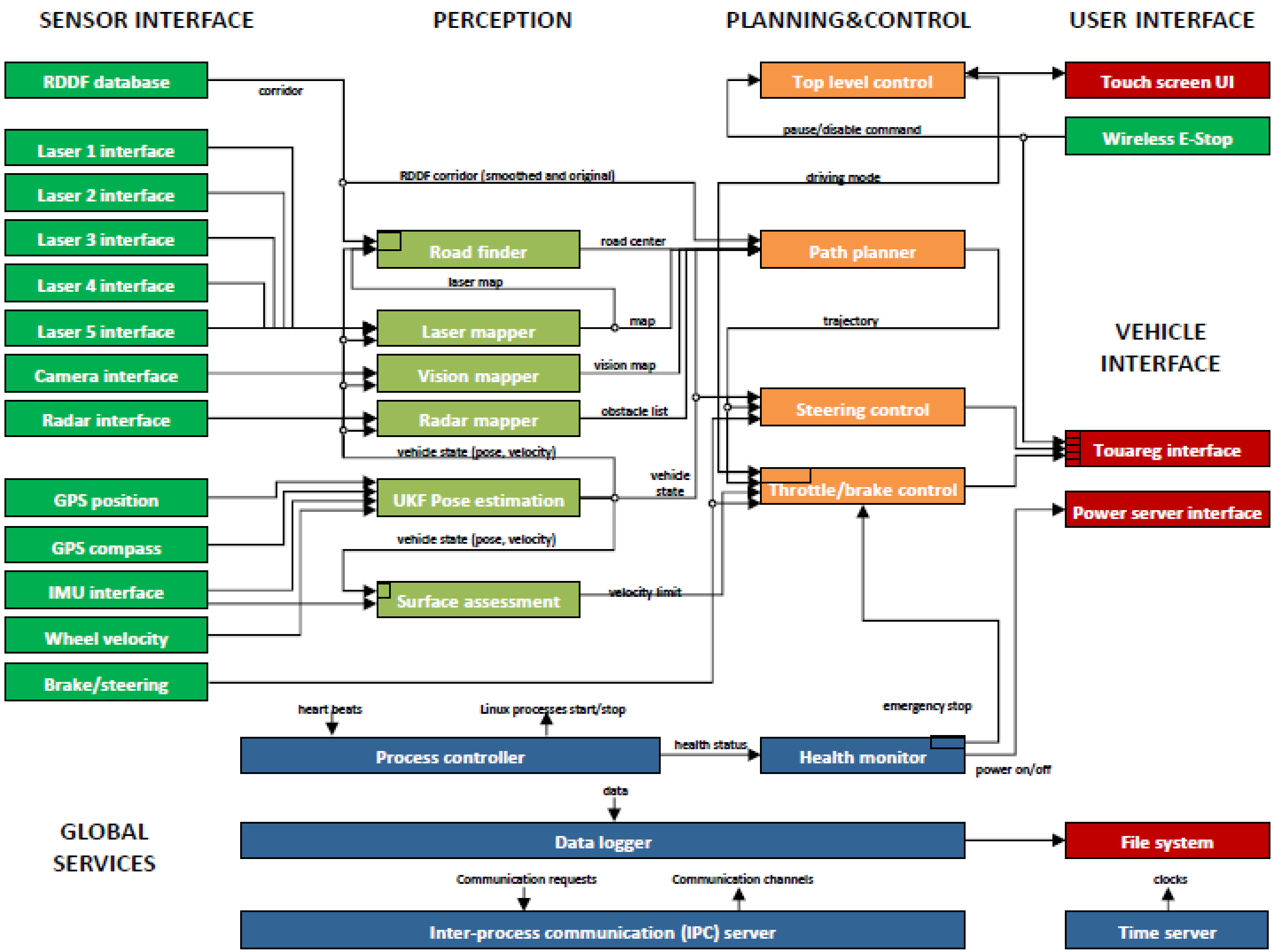


Example Architecture for Mobile Robot



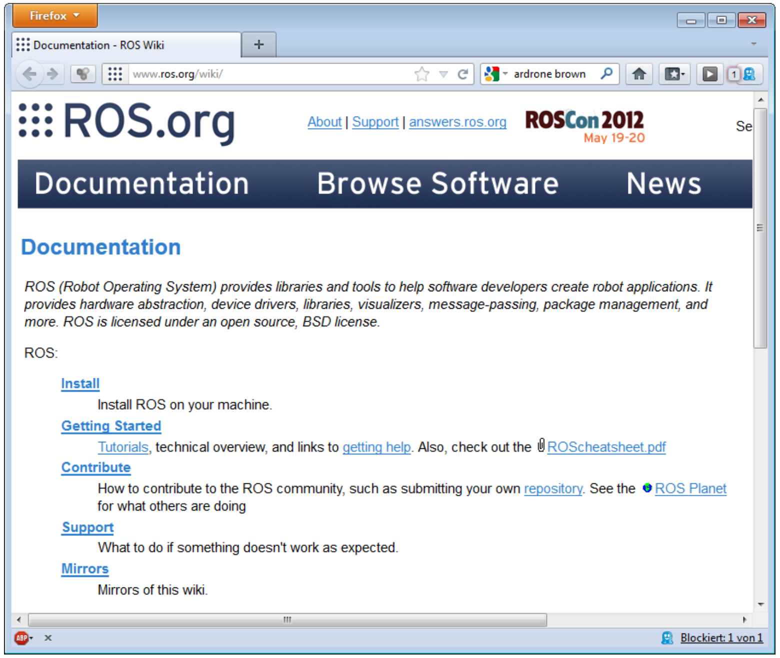


Stanley's Software Architecture



Robot Operating System (ROS)

- We will use ROS in the lab course
- http://www.ros.org/
- Installation instructions, tutorials, docs



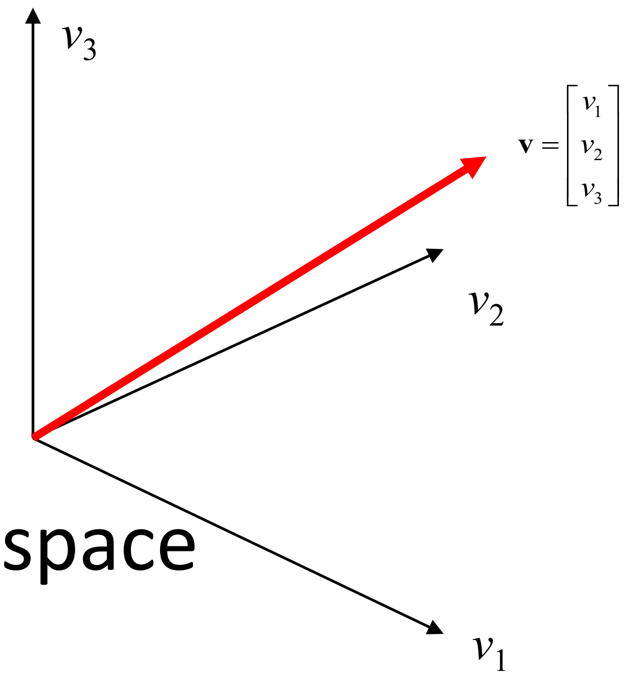
Short Notes on Linear Algebra

- Vector
- Vector Operations
  - Scalar Multiplication
  - Addition/Subtraction
  - Length/Normalization
  - Dot Product
  - Cross Product

- Matrix
- Types of Matrices
- Matrix Operations
  - Scalar Multiplication
  - Addition/Subtraction
  - Transpose
  - Determinant
  - Inverse
  - Square root
  - Jacobian / Derivative
  - Matrix Vector multiplication

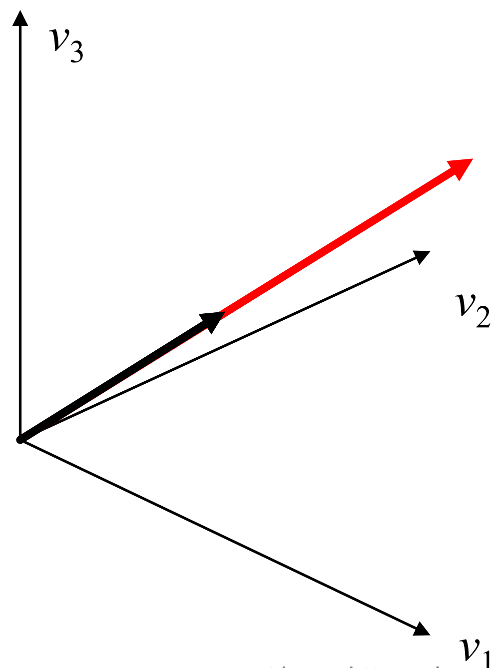
Vector

- Vector in R^n is an ordered set of n real numbers e.g. $V = [v_1, v_2, v_3]$ is in R^3
- $V = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$ is a column vector
- $V = [v_1 \quad v_2 \quad v_3]$ is a row vector
- Think of a vector as a point or line in a n-dimensional space



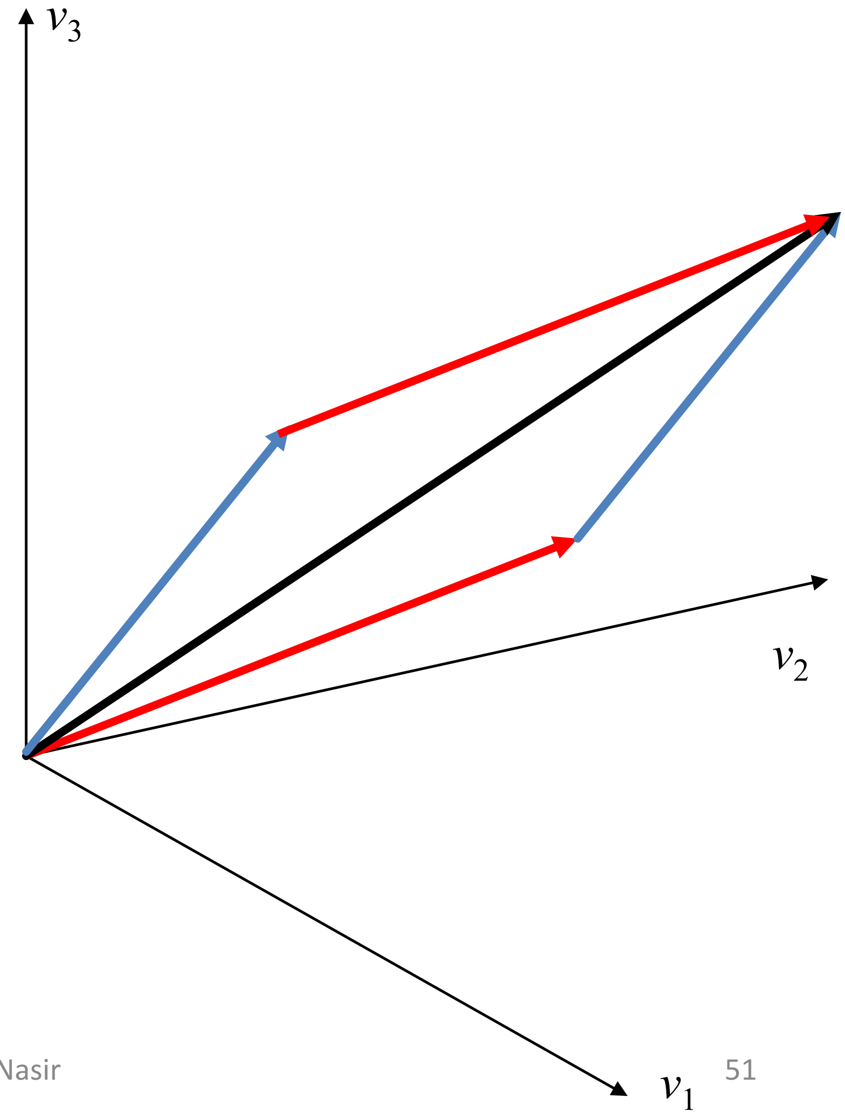
Vector Operations (Scalar Multiplication)

- Changes only the length but keeps the direction fixed
- $a \cdot [v_1 \quad v_2 \quad v_3] = [a \cdot v_1 \quad a \cdot v_2 \quad a \cdot v_3]$



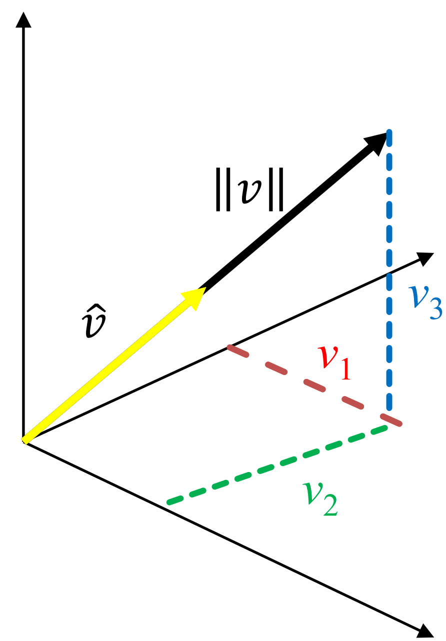
Vector Operations (Addition/Subtraction)

- V±W=U
- $\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \pm \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} u_1 \pm v_1 \\ u_2 \pm v_2 \\ u_3 \pm v_3 \end{bmatrix}$
- Vectors can be added or subtracted graphically using head and tail rule



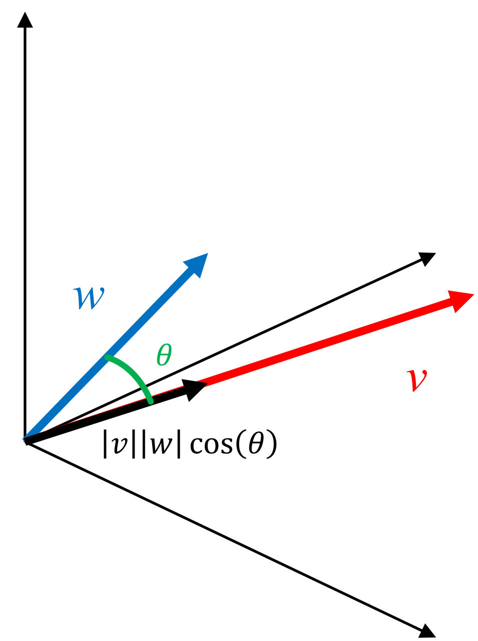
Vector Operations (Length/Normalization)

- If vector components are known then its magnitude or length can be determined
- $\|V\| = \sqrt{v_1^2 + v_2^2 + v_3^2}$
- Normalized or unit vector has a magnitude of 1, it is used for direction
- $\hat{V} = \frac{V}{\|V\|}$



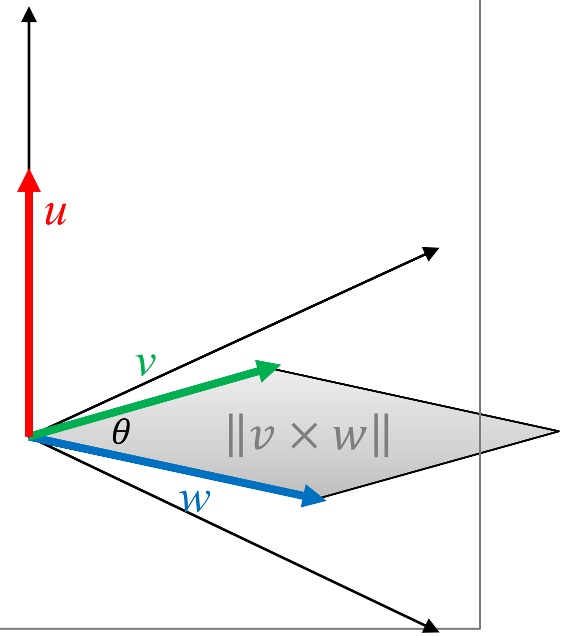
Vector Operations (Dot Product)

- The dot product measures to what degree two vectors are aligned in other words it can be used to calculate the angle between two vectors
- $V \cdot W = |V||W| \cos(\theta)$
- For orthogonal vectors $V \cdot W = 0$
- Magnitude is the dot product of the vector with itself
- $\|V\| = V^T \cdot V = \sum x_i \cdot x_i$



Vector Operations (Cross Product)

- Cross product of two vectors is a vector perpendicular to both vectors i.e.
- $U = V \times W$
- Magnitude of the cross product is the area of parallelogram i.e.
- $\|V \times W\| = \|V\| \|W\| \sin(\theta)$



Matrix

- Matrix is a set of elements, organized into rows and columns

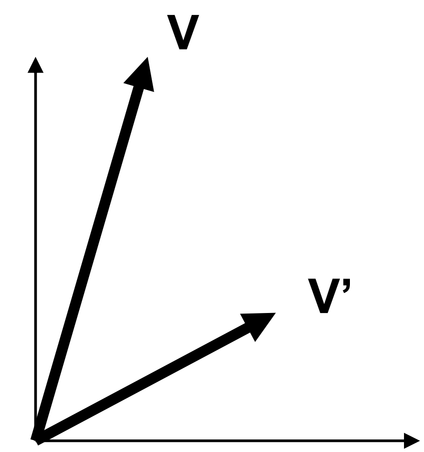
columns

rows

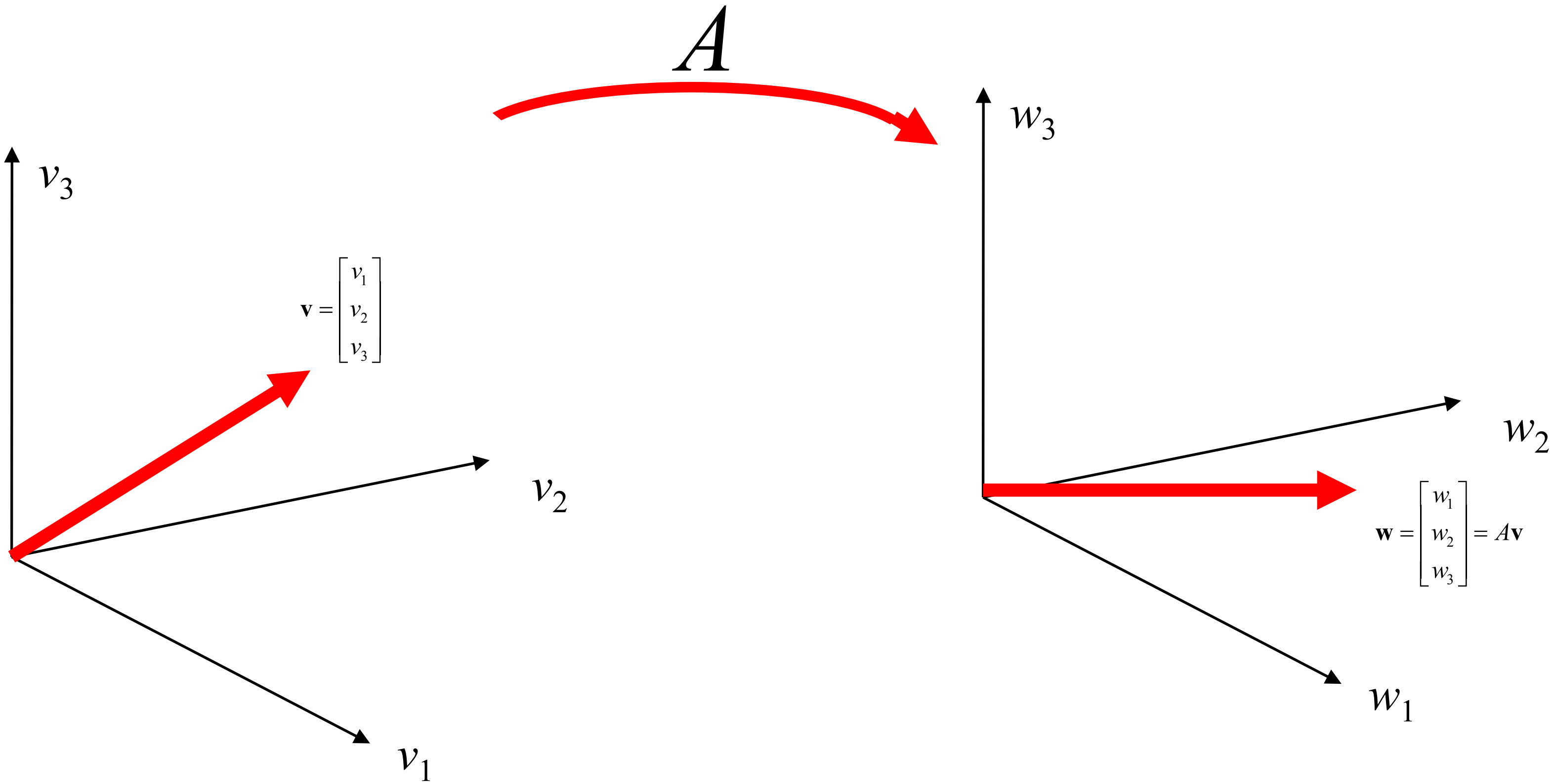
$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

- Think of a matrix as a transformation on a line/point or set of lines/points

$$\begin{bmatrix} x \\ y \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} x' \\ y' \end{bmatrix}$$

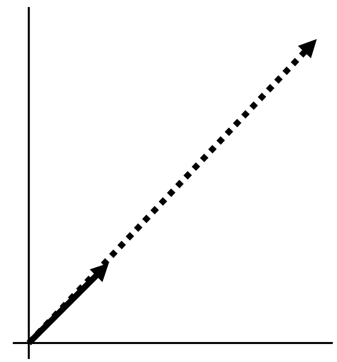


Matrices (Cont.)



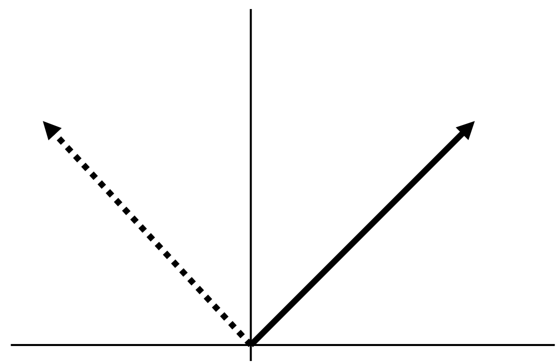
Matrices as linear transformations

$$\begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 5 \\ 5 \end{pmatrix}$$



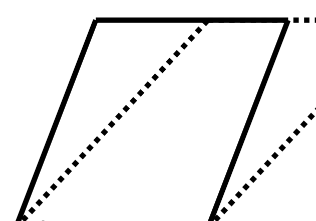
(stretching)

$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$



(rotation)

$$\begin{pmatrix} 1 & c \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x + cy \\ y \end{pmatrix}$$



(shearing)

Type of Matrices

$$\begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{pmatrix}$$ diagonal

$$\begin{pmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & f \end{pmatrix}$$ upper-triangular

$$\begin{pmatrix} a & b & 0 & 0 \\ c & d & e & 0 \\ 0 & f & g & h \\ 0 & 0 & i & j \end{pmatrix}$$ tri-diagonal

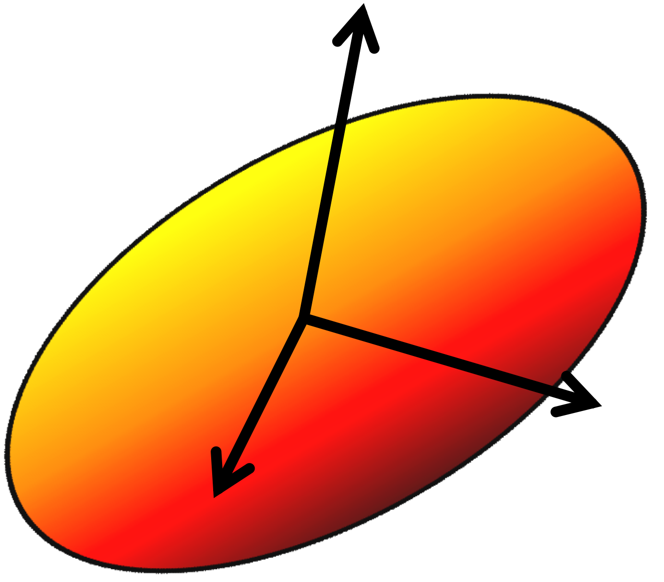
$$\begin{pmatrix} a & 0 & 0 \\ b & c & 0 \\ d & e & f \end{pmatrix}$$ lower-triangular

Matrix A is *symmetric* if $A = A^T$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$ I (identity matrix)

Type of Matrices
Positive(Semi) Definite Matrix

- If the matrix A is positive definite then the set of points, x, that satisfy $x'Ax = c$ where c>0 are on the surface of an n-dimensional ellipsoid centered at the origin



- Useful fact: Any matrix of form A^TA is positive semi-definite

Type of Matrices
Orthogonal/Orthonormal Matrix

1. Orthogonal matrices
   - A matrix is orthogonal if $P'P = PP' = I$
   - In this cases $P^{-1}=P'$.
   - Also the rows (columns) of *P* have length 1 **and** are orthogonal to each other

Orthogonal transformation preserve length and angles

Matrix Operation (Scalar Multiplication)

- Let $A = (a_{ij})$ denote an $n \times m$ matrix and let c be any scalar. Then cA is the matrix

$$cA = \left(ca_{ij}\right) = \begin{bmatrix} ca_{11} & ca_{12} & \cdots & \ldots \\ ca_{21} & ca_{22} & \cdots & \ldots \\ \vdots & \vdots & \ddots & \vdots \\ ca_{m1} & ca_{m2} & \cdots & \ldots \end{bmatrix}$$

Matrix Operation (Addition/Subtraction)

Let $A = (a_{ij})$ and $B = (b_{ij})$ denote two $n \times m$ matrices Then the sum, $A + B$, is the matrix

$$A+B=\left(a_{ij}+b_{ij}\right)=\begin{bmatrix} a_{11}+b_{11} & a_{12}+b_{12} & \cdots & \dots & b_{1n} \\ a_{21}+b_{21} & a_{22}+b_{22} & \cdots & \dots & b_{2n} \\ \vdots & \vdots & \ddots & \vdots & \\ a_{m1}+b_{m1} & a_{m2}+b_{m2} & \cdots & \dots & b_{mn} \end{bmatrix}$$

The dimensions of A and B are required to be both $n \times m$.

Matrix Operation (Transposition)

- Consider the $n \times m$ matrix, A

$$A = \left(a_{ij}\right) = \begin{bmatrix} a_{11} & a_{12} & \cdots & \cdots \\ a_{21} & a_{22} & \cdots & \cdots \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & \cdots \end{bmatrix}$$

then the $m \times n$ matrix, A' (also denoted by A^T)

$$A' = \left(a_{ji}\right) = \begin{bmatrix} a_{11} & a_{21} & \cdots & \cdots \\ a_{12} & a_{22} & \cdots & \cdots \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & \cdots \end{bmatrix}$$

Matrix Operation (Determinant)

- Used for inversion
- If det(A) = 0, then A has no inverse

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a\begin{vmatrix} e & f \\ h & i \end{vmatrix} - b\begin{vmatrix} d & f \\ g & i \end{vmatrix} + c\begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

$$= a(ei - fh) - b(di - fg) + c(dh - eg)$$

$$= aei + bfg + cdh - ceg - bdi - afh.$$

- Multiplication of Eigen values

Matrix Operation (Inversion)

- A^{-1} does not exist for all matrices A
- A^{-1} exists only if A is a square matrix and $|A| \neq 0$
- If A^{-1} exists then the system of linear equations has a unique solution

$$A\vec{x} = b$$

$$\vec{x} = A \quad b$$

$$A^{-1} = \frac{1}{|A|} \begin{bmatrix} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} & \begin{vmatrix} a_{13} & a_{12} \\ a_{33} & a_{32} \end{vmatrix} & \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix} \\ \begin{vmatrix} a_{23} & a_{21} \\ a_{33} & a_{31} \end{vmatrix} & \begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix} & \begin{vmatrix} a_{13} & a_{11} \\ a_{23} & a_{21} \end{vmatrix} \\ \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} & \begin{vmatrix} a_{12} & a_{11} \\ a_{32} & a_{31} \end{vmatrix} & \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \end{bmatrix}.$$

Matrix Operation (Square Root)

- Matrix B is said to be square root of A if BB=A
- In the Unscented Kalman Filter (UKF) the square root of the state error covariance matrix is required for the unscented transform which is the statistical linearization method used

Matrix Operations (Jacobian/Derivative)

Let $\vec{x}$ denote a $p \times 1$ vector. Let $f(\vec{x})$ denote a function of the components of $\vec{x}$.

$$\frac{df(\vec{x})}{d\vec{x}} = \begin{bmatrix} \frac{df(\vec{x})}{dx_1} \\ \vdots \\ \frac{df(\vec{x})}{dx_p} \end{bmatrix}$$

Matrix Operation (Matrix-Vector Multiplication)

- Matrix is like a function that transforms the vectors on a plane
- Matrix operating on a general point => transforms x- and y-components
- *System of linear equations*: matrix is just the bunch of coeffs !

- x’ = ax + by
- y’ = cx + dy

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x' \\ y' \end{bmatrix}$$

Matrix Operation (Matrix-Matrix Multiplication)

$$\mathbf{L} = \mathbf{M} \cdot \mathbf{N}$$

$$\begin{bmatrix} l_{11} & l_{12} & l_{13} \\ l_{21} & l_{22} & l_{23} \\ l_{31} & l_{32} & l_{33} \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix} \cdot \begin{bmatrix} n_{11} & n_{12} & n_{13} \\ n_{21} & n_{22} & n_{23} \\ n_{31} & n_{32} & n_{33} \end{bmatrix}$$

$$l_{12} = m_{11}n_{12} + m_{12}n_{22} + m_{13}n_{32}$$

Short Notes on 2D/3D Geometry

Let's apply some concepts of matrix algebra

- 2D/3D Points
- Line
- Plane
- Transformation
- Rotation Matrix
- Axis Angle / Quaternion
- Euler Angles

2D and 3D Points

- Consider 2D/3D points as column vector

$$V = \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

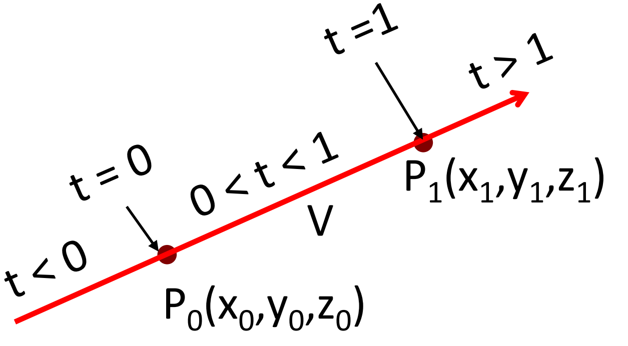
- Transformations are represented as 4x4 matrix

$$A = \begin{bmatrix} r_{11} & r_{11} & r_{11} & x \\ r_{11} & r_{11} & r_{11} & y \\ r_{11} & r_{11} & r_{11} & z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

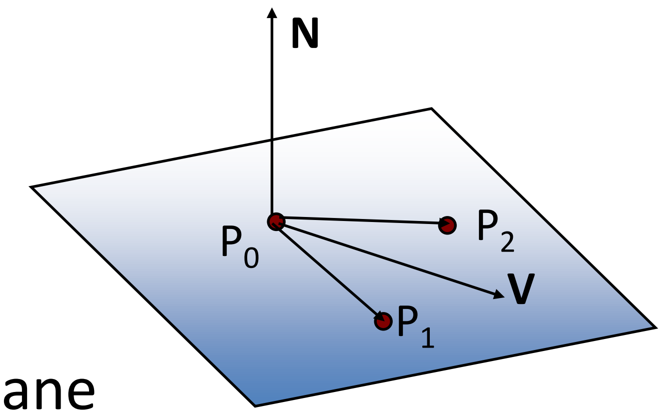
2D/3D Line

$$L: \quad \begin{aligned} x &= x_0 + (x_1 - x_0) \times t \\ y &= y_0 + (y_1 - y_0) \times t \\ z &= z_0 + (z_1 - z_0) \times t \end{aligned} \qquad 0 \le t \le 1$$

$$L = P_0 + t(P_1 - P_0)$$



3D Plane



- Ways of defining a plane
  1. 3 points P_0, P_1, P_2 on the plane
  2. Plane Normal **N** & P_0 on plane
  3. Plane Normal **N** & a vector **V** on the plane

Plane Passing through P_0, P_1, P_2

$$\overline{N} = \overrightarrow{P_0P_1} \times \overrightarrow{P_0P_2} = A\hat{i} + B\hat{j} + C\hat{k}$$

if P(x, y, z) is on the plane

$$\overline{N} \bullet \overrightarrow{P_0P} = 0$$

$$\Rightarrow (A\hat{i} + B\hat{j} + C\hat{k}) \bullet \left[(x - x_0)\hat{i} + (y - y_0)\hat{j} + (z - z_0)\hat{k}\right] = 0$$

$$\Rightarrow Ax + By + Cz + D = 0$$

$$\textit{where} \quad D = -(Ax_0 + By_0 + Cz_0)$$

Transformations

- **Transformation** – is a function that takes a point (or vector) and maps that point (or vector) into another point (or vector).
- **Line**: Can be transformed by transforming the end points
- **Plane**:(**described by 3-points**) Can be transformed by transforming the 3-points
- **Plane**:(**described by a point and Normal**) Point is transformed as usual. Special treatment is needed for transforming Normal

3D Transformation

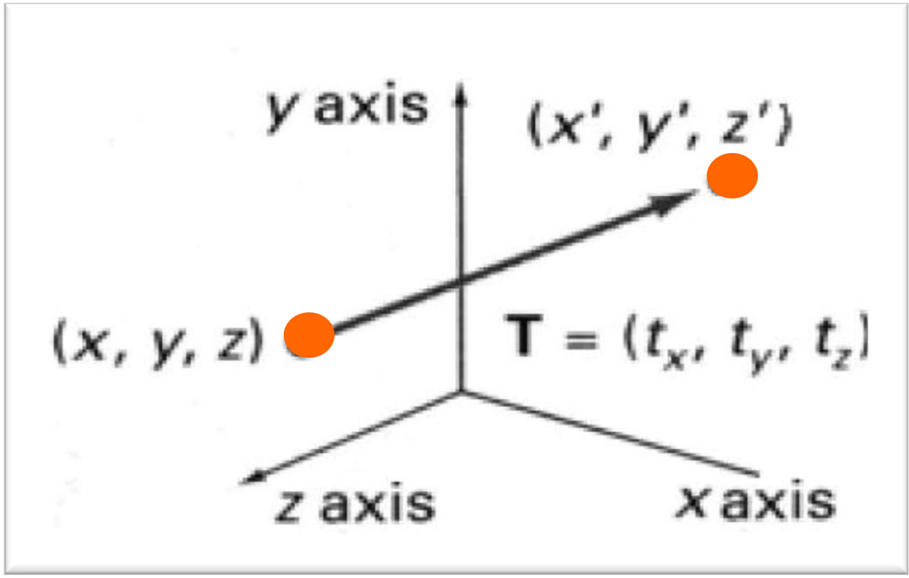
- A coordinate transformation of the form:

$$x' = a_{xx}x + a_{xy}y + a_{xz}z + b_x,$$
$$y' = a_{yx}x + a_{yy}y + a_{yz}z + b_y,$$
$$z' = a_{zx}x + a_{zy}y + a_{zz}z + b_z,$$

is called a 3D ***affine transformation***.

$$\begin{pmatrix} x' \\ y' \\ z' \\ w \end{pmatrix} = \begin{pmatrix} a_{xx} & a_{xy} & a_{xz} & b_x \\ a_{yx} & a_{yy} & a_{yz} & b_y \\ a_{zx} & a_{zy} & a_{zz} & b_z \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

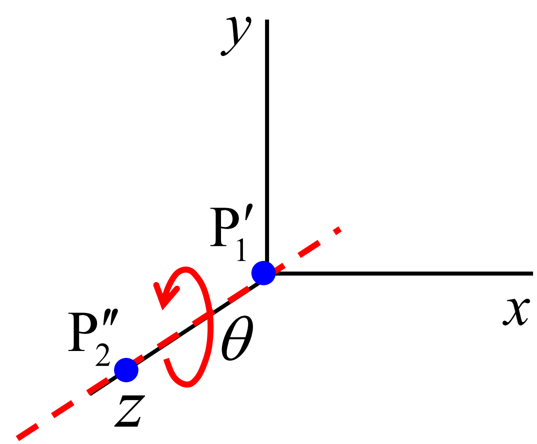
- The 4th row for affine transformation is always [0 0 0 1].
- Properties of affine transformation:
  - translation, scaling, shearing, rotation (or any combination of them)
  - Lines and planes are preserved.
  - parallelism of lines and planes are also preserved, but not angles and length.



Rotation Matrix

Let R be

$$R = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} = \begin{bmatrix} R_x.x & R_x.y & R_x.z \\ R_y.x & R_y.y & R_y.z \\ R_z.x & R_z.y & R_z.z \end{bmatrix}$$



R is Rigid-body Transform

i) $\vec{R_x}, \vec{R_y}, \vec{R_z}$ *are unit vectors*

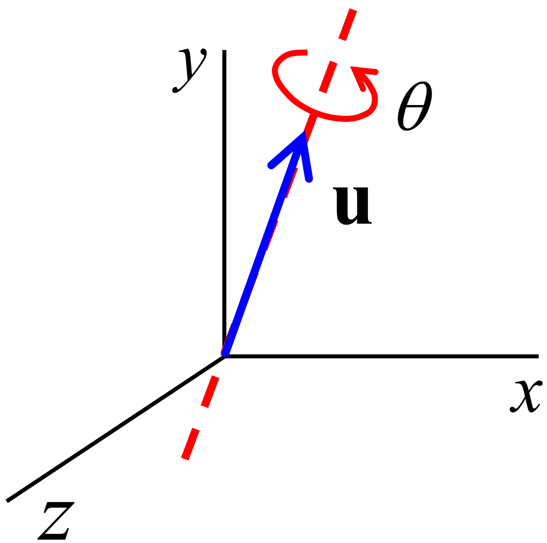
ii) $\vec{R_x}, \vec{R_y}, \vec{R_z}$ *are perpendicular to each other*

$$\mathbf{R}_z(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Note: $R_x.x \Rightarrow x$ *component of vextor* $\vec{R_x}$

Axis/Angle Rotation

Rotate a point position $\mathbf{p} = (x, y, z)$ about the unit vector $\mathbf{u}$.



Quarternion representation:

Rotation: $q = \left(\cos\frac{\theta}{2} \;, \mathbf{u}\sin\frac{\theta}{2} \right)$

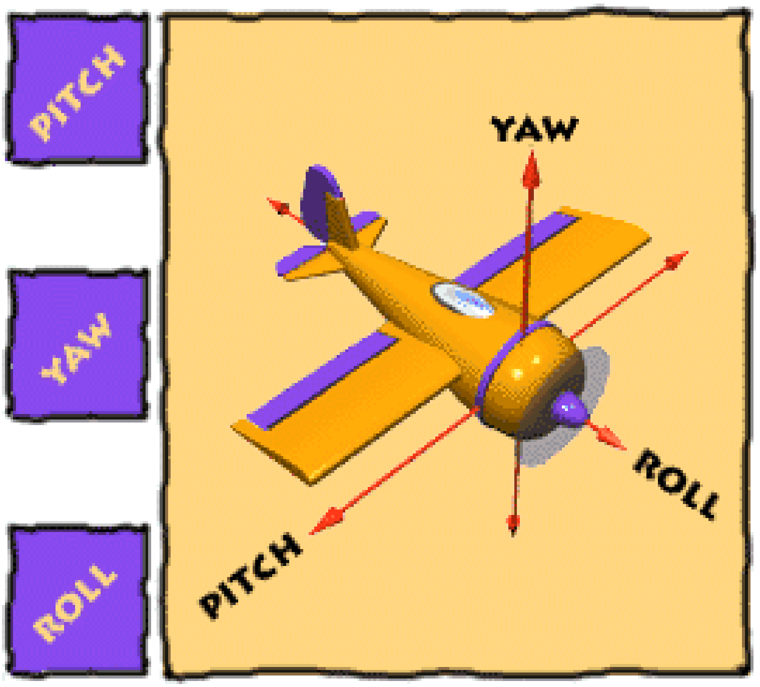
Position: $\mathbf{P} = (0, \mathbf{p}), \quad \mathbf{p} = (x, y, z)$

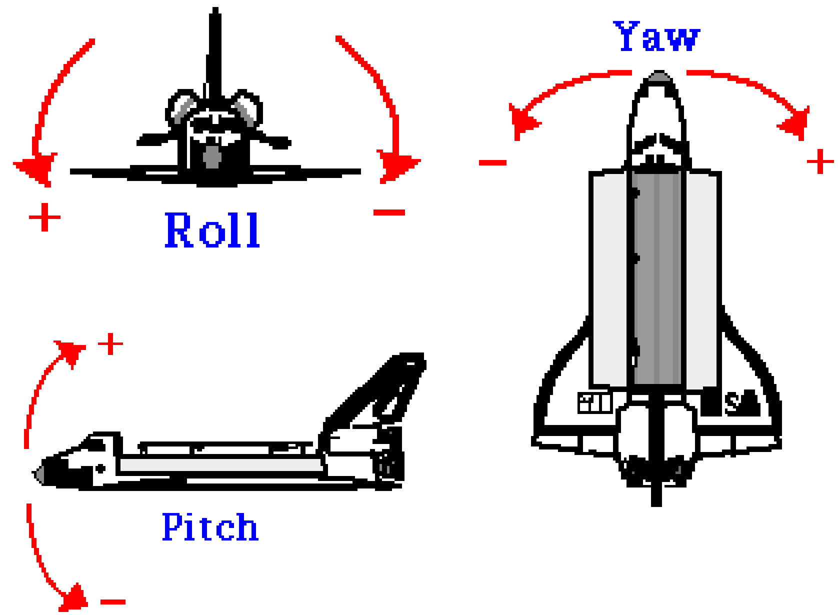
Rotation of **P** is carried out with the quarternion operation:

$$\mathbf{P}' = q\mathbf{P}q^{-1} = \left(0, s^2\mathbf{p} + \mathbf{v}(\mathbf{p}\cdot\mathbf{v}) + 2s(\mathbf{v}\times\mathbf{p}) + \mathbf{v}\times(\mathbf{v}\times\mathbf{p})\right)$$

Euler Angles

- Imagine three **lines** running through an airplane and intersecting at right angles at the airplane's center of gravity.



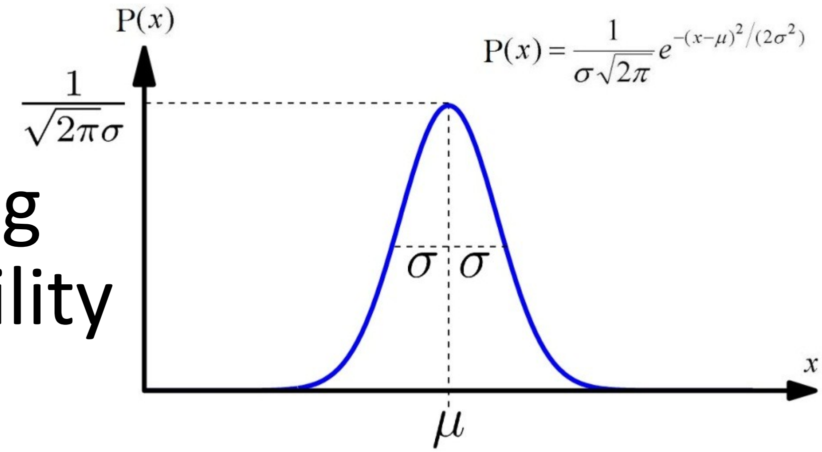


Recap of Probability Rules

- Discrete Random Variables
- Probability Density Functions
- Axioms of Probability Theory
- Joint and Conditional Probability
- Laws of Total Probability
  - Marginalization
  - Bayes Rule

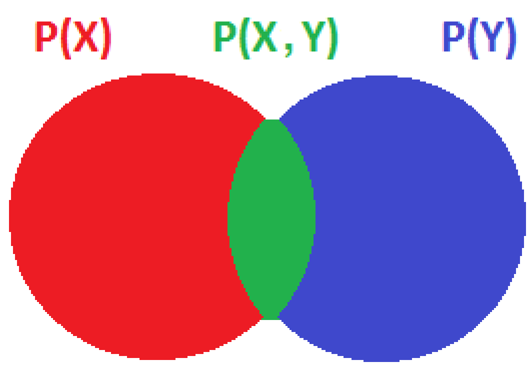
Random Variables (Discrete)

- X represetns a random variable
- X can take countable number of values: $\{x_1, x_2, ..., x_n\}$
- $P(\cdot)$ represents the probability function e.g. Gaussian, Uniform etc.
- $P(X = x_i)$ or $P(x_i)$ is the probability of occurring event x_i using the probability function $P(\cdot)$



Axioms of Probability Theory

- $0 \leq P(X) \leq 1$
- $P(TRUE) = 1$
- $P(FALSE) = 0$
- $P(X \cup Y) = P(X) + P(Y) - P(X \cap Y)$
- $P(\neg X) = 1 - P(X)$
- $\sum_x P(x_i) = 1$



Joint and Conditional Probability

- Joint Probability
  - $P(X = x \ and \ Y = y) = P(x \cap y) = P(x, y)$
  - If X and Y are independent $P(x, y) = P(x) \cdot P(y)$
- Conditional Probability
  - $P(x|y) = \frac{P(x,y)}{P(y)}$ or $P(x, y) = P(x|y) \cdot P(y)$
  - If X and Y are independent $P(x|y) = P(x)$

Law of Total Probability Marginalization and Bayes Formula

- Law of Total Probability
  - $P(x) = \sum_y P(x|y) \cdot P(y)$
- Marginalization
  - $P(x) = \sum_y P(x,y)$
- Bayes Formula
  - $P(x,y) = P(x|y) \cdot P(y) = P(y|x) \cdot P(x)$

$$\Rightarrow P(x|y) = \frac{P(y|x) \cdot P(x)}{P(y)} = \frac{expected \cdot piror}{measurement}$$

Summary

- Course Introduction
- Introduction to mobile robotics
- Review of basic concepts
  - Algebra
    - Vectors
    - Matrices
  - Geometry
    - Points, Lines, Plane
    - Transformations, Rotation Matrix, Quaternion, Euler Angles
  - Probability
    - Discrete random variables
    - Axioms and laws

Questions